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SUPERCONDUCTIVITY IN TWO-BAND MODEL BY RENORMALIZATION GROUP APPROACH

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We investigate the superconductivity in a two-band model by using the renormalization group approach. We discuss the superconducting gap function in the two-band model.

1. Theoretical Background

The concept of two-band superconductivity has been introduced, and the possibility of the superconductivity has been discussed by many groups.[1–3] Konsin and his coworkers have studied superconducting properties of cooper oxides by using a two-band model.[4,5] Recently, we have developed the two-band model to more general formalism.[6–8] When Fermi energy level crosses two bands, we consider two Cooper pairs at their Fermi momentum. If the Cooper pair tunnel between their regions as a coupled state, the tunneling more stabilizes the order of Cooper pair than that of a single band model.[9–12] Very recently, Kondo has also presented a theory for multi-band superconductivity.[13,14] In this study, we present a simple model for two-band superconductivity by using a renormalization group approach and discuss two gap functions in relation to more general model.

In this section, we briefly summarize a two-band model for the superconductivity and introduce a renormalization group approach.[15–17]

We consider a Hamiltonian for two bands i and j written as

$$H = H_0 + H_{\rm int}\,, \tag{1}$$

where

$$H_0 = \sum_{\boldsymbol{k},\sigma} \left[[\epsilon_i(\boldsymbol{k}) - \mu]\, a^\dagger_{i\boldsymbol{k}\sigma} a_{i\boldsymbol{k}\sigma} + [\epsilon_j(\boldsymbol{k}) - \mu]\, a^\dagger_{j\boldsymbol{k}\sigma} a_{j\boldsymbol{k}\sigma} \right], \tag{2}$$

$$\begin{aligned} H_{\rm int} = \frac{1}{4} \sum_{\delta(\boldsymbol{p}_1+\boldsymbol{p}_2,\boldsymbol{p}_3+\boldsymbol{p}_4)} \sum_{\alpha,\beta,\gamma,\delta} & [g_{i1}(\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\gamma}\delta_{\beta\delta}) a^\dagger_{i\boldsymbol{p}_1\alpha} a^\dagger_{i\boldsymbol{p}_2\beta} a_{i\boldsymbol{p}_3\gamma} a_{i\boldsymbol{p}_4\delta} \\ & + g_{j1}(\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\gamma}\delta_{\beta\delta}) a^\dagger_{j\boldsymbol{p}_1\alpha} a^\dagger_{j\boldsymbol{p}_2\beta} a_{j\boldsymbol{p}_3\gamma} a_{j\boldsymbol{p}_4\delta} \\ & + g_2(\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\gamma}\delta_{\beta\delta}) a^\dagger_{i\boldsymbol{p}_1\alpha} a^\dagger_{i\boldsymbol{p}_2\beta} a_{j\boldsymbol{p}_3\gamma} a_{j\boldsymbol{p}_4\delta} \\ & + g_2(\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\gamma}\delta_{\beta\delta}) a^\dagger_{j\boldsymbol{p}_1\alpha} a^\dagger_{j\boldsymbol{p}_2\beta} a_{i\boldsymbol{p}_3\gamma} a_{i\boldsymbol{p}_4\delta}], \end{aligned} \tag{3}$$

where $a^\dagger_{i\boldsymbol{p}\sigma}(a_{i\boldsymbol{p}\sigma})$ is the creation (annihilation) operator corresponding to the excitation of electrons (or holes) in ith band with spin σ and momentum $\boldsymbol{p}$. μ is the chemical potential. g_{i1} and g_{j1} represent the ith and jth intraband two-particle normal scattering processes, respectively. g_2 indicates the intraband two-particle umklapp scattering.

2. Renormalization Group Approach

The Dyson equation is invariant under a multiple renormalization of Green's function and coupling parameters g. From this invariance for a scaling procedure, we obtain differential equations for the coupling parameters and the external vertex of Cooper pair:

$$y\frac{\partial}{\partial y}\tilde{g}_i(y,u,g) = \frac{\partial}{\partial \xi}\tilde{g}_i(\xi, u/y, \tilde{g}(t,u,g))|_{\xi=1}\,, \tag{4}$$

$$y\frac{\partial}{\partial y}\ln\Lambda(y,u,g) = \frac{\partial}{\partial \xi}\ln\Lambda(\xi, u/y, \tilde{g}(t,u,g))|_{\xi=1}\,, \tag{5}$$

where y and u are parameters with the dimension of energy. g means the set of original coupling, and Λ is the external vertex.

To solve Eqs. (4) and (5), we estimate the right-hand side of Eq. (4) by using the perturbation theory. When we consider the lowest order correction to the vertex for Cooper pair, we obtain

$$\begin{pmatrix} \tilde{g}_{i1} \\ \tilde{g}_{j1} \end{pmatrix} = \begin{pmatrix} g_{i1} \\ g_{j1} \end{pmatrix} + \begin{pmatrix} -g_{i1}^2 & -g_2^2 \\ -g_2^2 & -g_{j1}^2 \end{pmatrix} \begin{pmatrix} L_i \\ L_j \end{pmatrix}, \tag{6}$$

$$\begin{pmatrix} \tilde{g}_2 \\ \tilde{g}_2 \end{pmatrix} = \begin{pmatrix} g_2 \\ g_2 \end{pmatrix} + \begin{pmatrix} -g_{i1}g_2 & -g_2 g_{j1} \\ -g_2 g_{i1} & -g_{j1}g_2 \end{pmatrix} \begin{pmatrix} L_i \\ L_j \end{pmatrix}, \tag{7}$$

where L_i and L_j correspond to the polarization function for each intraband. When $\epsilon_i(\boldsymbol{k}) = t_i k^2$, and $\epsilon_j(\boldsymbol{k}) = t_j k^2$, for the special case $\omega = 0$, $\boldsymbol{k} = 0$, the above functions L_i and L_j become

$$L_i = \frac{1}{2\pi}\sqrt{u_i/\xi} - \sqrt{u_i/u_i'}\,, \tag{8}$$

$$L_j = \frac{1}{2\pi}\sqrt{u_j/\xi} - \sqrt{u_j/u_j'}\,, \tag{9}$$

where u_i (u_j) and u_i' (u_j') are non-dimensional functions expressed by the chemical potential, cut-off energy, the top energy of jth band, and the density of state for ith (jth) band.

Next, we consider a first order response function for singlet Cooper pair. The first order vertex function Λ for ith and jth bands can be written as

$$\begin{pmatrix}\Lambda_i\\ \Lambda_j\end{pmatrix} = \begin{pmatrix}1\\ 1\end{pmatrix} + \begin{pmatrix}-g_{i1} & -g_2\\ -g_2 & -g_{j1}\end{pmatrix}\begin{pmatrix}L_i\\ L_j\end{pmatrix}. \tag{10}$$

For simplicity, here and hereafter we assume $g_{i1} = g_{j1} = g_1$. From Eqs. (4), (6) and (7), we obtain differential equations written as

$$\frac{\partial}{\partial x}\tilde{g}_1 = -(\tilde{g}_1^2 + \tilde{g}_2^2)\,, \tag{11}$$

$$\frac{\partial}{\partial x}\tilde{g}_2 = -2\tilde{g}_1\tilde{g}_2\,. \tag{12}$$

In similar way, using Eqs. (5) and (10), we obtain differential equations written as

$$\frac{\partial}{\partial x}\ln\Lambda_+ = -\tilde{g}_1 - \tilde{g}_2\,, \tag{13}$$

where $\Lambda_+ = (\Lambda_i + \Lambda_j)/2$.

3. Results and Discussion

In the previous section, we have derived basic equations of Eqs. (11)–(13) to find the low-temperature phases. For the special case of $g_2 = 0$, we obtain an analytic solution and find that the superconducting phase appears only when the intraband interaction g_1 is negative. In the case of the traditional superconductivity such as BCS theory, it is necessary that effective electron-electron interaction is negative ($g_1 < 0$) for realizing superconductivity. The present result agrees with that of the traditional theory for superconductivity expressed by one-band model. On the other hand, for the case of $g_2 \neq 0$, we can find superconductivity in the case of $g_2 < -g_1$.

In the case $g_{i1} \neq g_{j1}$, after diagonalization of the matrix in the righthand side of Eqs. (6), (7), and (10), the above equations of Eqs. (11)–(13) are rewitten. In this case, we can consider the possibility of two gap functions and expect interesting phase diagrams for two-band superconductivity. Superconductivity of MgB_2 may be a candidate to treat along this scheme. Phase diagrams for CDW, SDW,

singlet superconductivity derived from more general Hamiltonian will be presented elsewhere.

In conclusion, we derive a renormalization equations for two-band superconductivity and discuss two gap functions in the two-band superconductivity. In the framework of this model, the present results predict that it is impotant that g_1 is attractive for superconductivity.

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